Assignment 2: Sensitivity Analysis

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# Rate sensitivity analysis

*Determine to which rates (inflows and outflows)* ***your model*** *is most sensitive.*

* *Select the output variables and the metric you’ll use to evaluate the sensitivity of these variables. Explain and justify your choices.*
* *Conduct a sensitivity analysis in which you vary all the rates in your model by + and – 1%. Show the resulting behavior plots in your report.*
* *Determine the rates that cause the highest sensitivity in each put variable.*
* *Discuss your results with reference to the model structure and model robustness.*
* *Select 5 rates and repeat the procedure with + and – 10% perturbation. Discuss if the results change or not.*

The output variables on which sensitivity was observed were chosen by their importance in the main structure of the model. In one of the initial model building phases we constructed a causal loop diagram (figure 1) which shows the system with its most important variables on a high level.

All variables in the causal loop diagram were chosen as output variables for the sensitivity analysis. These are:

* Water demand
* Infrastructure shortage
* Available funds
* Available infrastructure
* Staff availability
* Infrastructure maintenance

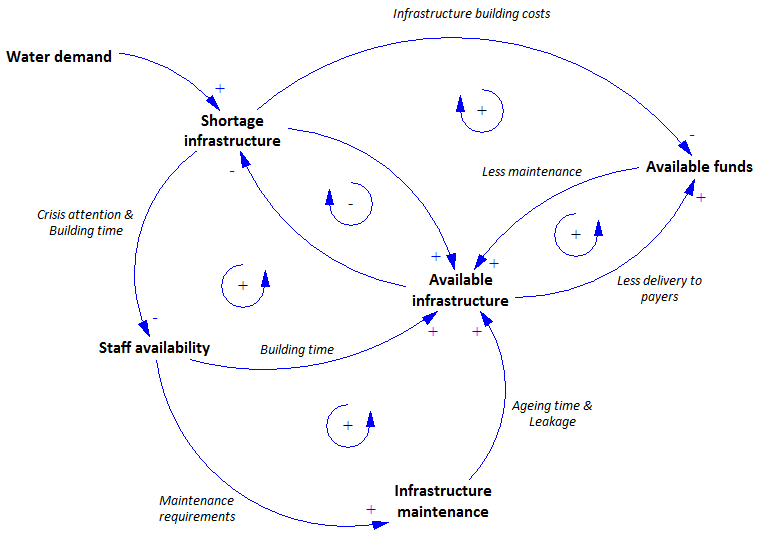


Figure 1: High level causal loop diagram

When performing the sensitivity analysis by varying the rates by 1% we noticed that there was no visual effect on these variables since the variation is simply too small. All the lines of the plot were drawn over one another and were indistinguishable. Therefore, the 1% variation plots are not shown in this report. Later on, the 10% variation plots will be shown resulting from the 5 chosen rates.

For determining the sensitivity the following formula was used:

This sensitivity was numerically estimated by using the trapezoid rule which gives the following formula:

The actual calculation of the sensitivity value was done using a script we wrote in R. This script takes as input a directory containing csv result files from Vensim and the column names of the output variables you’re interested in, and returns a matrix with as the rows the experiment and as columns the output variables. The R script is added as an appendix to this document.

The results of the sensitivity analysis on each of the 6 output variables are shown in table 1. From this table it becomes clear that the 5 variables which show the largest sensitivity on the model are:

* Infrastructure aging
* Staff leaving
* Staff hiring
* Maintenance expenditures
* Income flow

For each of these variables an additional sensitivity analysis is done but this time varying the rates 10% instead of 1%. These results are also shown in table 1. A plot matrix of the effect of the 5 most sensitive rates on the output variables is shown in figure 2.

Results related to model robustness

We expect that the total water demand is only influenced by rates in the population system. The sensitivity analysis clearly shows this since the relative sensitivity resulting from all rate changes in other systems is zero for the total water demand.

As stated before, the results of the sensivity analysis with rate changes of -1% and +1% are so minimal that they are not visible in graphs. This is concurrent with the results of table 1, as all relative sensitivity values are very small, ranging from 0 to 2,242. Although this test is only done with very small rate changes, this is a first test to see whether the model will show the same behaviour, which it indeed does. Also numerical sensitivity is almost non-existent. So far, the model looks to be robust.

More conclusions can be made from the -10% and +10% rate changes. The sensitivity results of these rate changes are shown in table 1 as well as in appendix B where the different results are visualized in graphs. Increasing or decreasing the rates by 10% does not give massively different results. No behavioural sensitivity is observed and the sensitivity values in table 1 are roughly increased 10 times which is within line of expectation. The only rate change which results in a ‘significant’ relative sensitivity in this test is the -10% change in the infrastructure aging rate which results in a relative sensitivity of 32,798 for the practical infrastructure capacity variable which is considerably higher than the other relative sensitivity values. To get more insight in the effects of this rate change on this output value, the corresponding graph is checked in appendix B. This graph, as all the other graphs, shows no behavioural sensitivity. Furthermore, the numerical sensitivity seems to be insignificant. We conclude that the model is robust.



Table 1: Relative sensitivity of output variables resulting from rate changes of -10%, -1%, +1% and +10%

# Table function sensitivity analysis

*For at least one table function in your model, explore how sensitive your model is to changing the shape of the function by using a distortion function.*

* *Select the output variables, a distortion function type and the uncertainty ranges assigned to the parameters of this distortion function.*
* *Explain and justify your choices.*
* *Conduct a multi-variate automated sensitivity analysis and discuss the results in terms of the type and extent of the resulting sensitivity.*
* *Conduct uni-variate sensitivity analyses with at least two different values of the distortion function parameters.*
* *Discuss how these changes in the distortion function parameters affect (i) the shape of the distortion function, (ii) the shape of the table function and (iii) the model behavior.*

The lookup which is chosen for sensitivity analysis is the lookup which decreases staff productivity in case of increasing staff productivity.

This lookup is currently modelled as a follows: When staff occupancy is very low, the available staff will have a 50% increased productivity. However, as staff occupancy increases this productivity gradually drops to a minimum of 10% productivity when Kirkwood is 5 times understaffed (Figure 3).

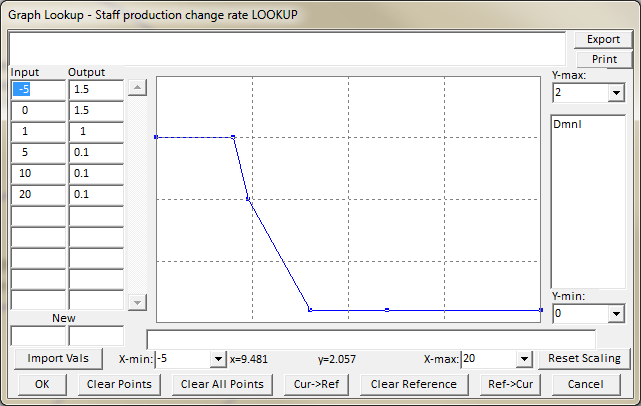


Figure 3: Initial lookup

This lookup function directly influences the total staff production available which in turn can greatly influence the infrastructure capacity. Therefore, these variables are chosen as output variables.

* Total production possible
* Practical infrastructure capacity
* Infrastructure shortage

The lookup function contains two aspects. The first aspect is the increase in productivity when staff occupancy is low, and the second is the decrease in productivity when staff occupancy is high. Both of these aspects should be tested using the distortion function.

Therefore, a double triangular distortion function will be used in order to both vary the productivity increase and decrease. The input range of the lookup function is from 0 to 5. The input will never be able to drop below zero since it is a rate, and for values higher than 5 the last value will be extrapolated. At staff occupancy of 1, there is no change in staff productivity. Therefore, the distortion function will be fixed at point (1, 1).

An example distortion function is shown in figure 4. This distortion function can be controlled using 4 parameters:

* Peak 1 height – Value in R0+
* Peak 1 position – Value between 0 and 1
* Peak 2 height – Value in R0+
* Peak 2 position – Value between 1 and 5

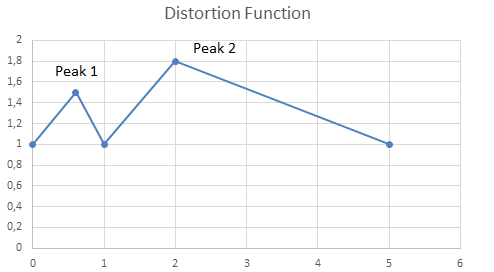


Figure 4: Example distortion function

This is modelled in Vensim as shown in figure x.

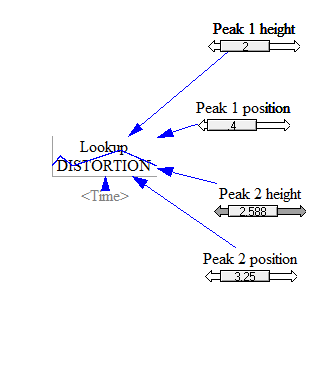


Figure 5: Vensim implementation

Results of multi-variate analysis







Figure 6: Multivariate sensitivity analysis results

A multi-variate sensitivity analysis was performed by varying the four parameters of the distortion function as described in the previous section. For the peak heights, values between 0 and 5 were chosen. A peak height of 0 for peak 1 results in there being no effect on staff productivity if staff occupancy is low. A peak height of 5 for peak 1 will result in much higher increased productivity at low staff occupancy. The height of peak 2 has the same effect for high values of staff occupancy.

Figure 5 shows the results of a multi-variate sensitivity analysis of the described lookup function on the three aforementioned output variables.

In the first 6 years of the system the staff occupancy is low. By varying the peak 1 height of the distortion function between 0 and 5 we expect the total production possible range for the first 6 years to be within 50% lower and 5 times higher than the baseline. Figure 5 clearly shows this.

For the period after these 6 years the staff occupancy increases drastically. If peak 2 height is reduced to zero we expect the total production possible to simply show a smooth decline due to staff leaving. This effect is shown in the upper right quadrant of the graph. However, if we increase the peak 2 height the effect of higher staff occupancy on production rates becomes much larger and we expect the total production possible to decrease much more rapidly than in the baseline. This is shown in the lower right quadrant.

The practical infrastructure capacity and infrastructure shortage are closely related and will therefore be discussed at once. At the start of the simulation there is an infrastructure surplus. For the first six years this surplus remains which results in there being no constraints on staff production. When infrastructure starts to drop below the water demand the staff will be unable to keep up with demand and shortages will rise. When the effect of high staff occupancy on the staff productivity is reduced we clearly see that the infrastructure capacity remains on the right level for a much longer period.

Univariate sensitivity analysis

For the univariate sensitivity analysis two scenarios will be explored. The first scenario is when decreased staff occupancy does not increase staff productivity. The second scenario is when increased staff occupancy does not decrease staff productivity. The results of this sensitivity analysis are shown in figure 6. The used distortion functions are shown in figure 7.

In scenario 1, we expect the initial staff production possible to be much lower than the baseline scenario since staff occupancy is low in the beginning but its effect is switched of. When staff occupancy becomes greater than 1, we expect scenario 1 to follow the base case. For scenario 2 this is entirely opposite. The initial period should follow the base case but the productivity after the first 6 years should drop sharply.

For the infrastructure capacity and infrastructure shortage we expect to see not much difference in the initial years of the simulation since there is initially an infrastructure surplus which means staff doesn’t influence these variables. As soon as there is an infrastructure shortage there is also a high staff occupancy. Therefore, scenario 1 will show the same behaviour as the base case. Scenario 2 shows slightly different behaviour since there is even less staff available. However, in the base case there is already very little staff available from 6 years onward so the difference is small.

Infrastructure shortage

Practical infrastructure capacity

Total production possible

Figure 7: Univatiate sensitivity analysis results

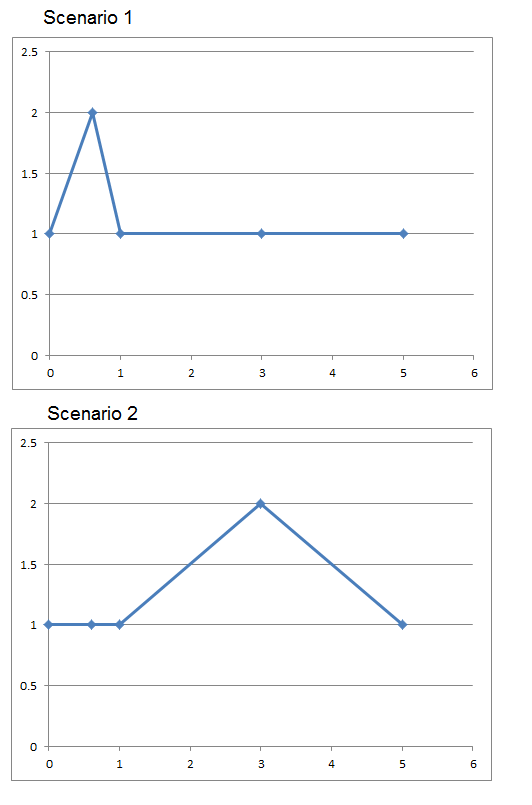


Figure 8: Lookup distortions

# Appendix A: R Script for sensitivity calculation



# Appendix B: Sensitivity plots

The following 3 pages show the results on the output variables of the sensitivity analysis with -10% and +10% changes in the 5 chosen rates from part 1 of this assignment visualized in graphs.

The following legend is used for all graphs.



Perturbation of the rate by +10%

Perturbation of the rate by -10%

Baseline

|  |  |  |
| --- | --- | --- |
| **Rate / Output** | **Water demand (Ml/year)** | **Infrastructure shortage (Ml/year)** |
| **Infrastructure aging** |  |  |
| **Hiring of staff** |  |  |
| **Staff leaving** |  |  |
| **Income flow** |  |  |
| **Maintenance expenditures** |  |  |

|  |  |  |
| --- | --- | --- |
| **Rate / Output** | **Practical infrastructure capacity (Ml/year)** | **Available funds for maintenance (kZAR)** |
| **Infrastructure aging** |  |  |
| **Hiring of staff** |  |  |
| **Staff leaving** |  |  |
| **Income flow** |  |  |
| **Maintenance expenditures** |  |  |

|  |  |  |
| --- | --- | --- |
| **Rate / Output** | **Total production possible (Ml/year)** | **% Infrastructure maintained (Dmnl)** |
| **Infrastructure aging** |  |  |
| **Hiring of staff** |  |  |
| **Staff leaving** |  |  |
| **Income flow** |  |  |
| **Maintenance expenditures** |  |  |